Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
International Journal of Solids and Structures 42 (2005) 4192–4203
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Thermo-elastic interaction without energy dissipation in an infinite solid with distributed periodically varying heat sources

S.K. Roychoudhuri, Partha Sharathi Dutta *

Department of Mathematics, University of Burdwan, Golapbag, Burdwan 713104, West Bengal, India

Received 13 May 2004; received in revised form 15 December 2004
Available online 26 January 2005

Abstract

The paper deals with the thermo-elastic interactions due to distributed periodically varying heat sources in a homogeneous, isotropic, unbounded elastic medium in the context of the theory of thermo-elasticity without energy dissipation. Closed form solutions for displacement, temperature, stress and strain are derived by using Laplace transform on time and then Fourier transform on space. It reveals that the interactions consist of two coupled modified dilatational and thermal waves modified by finite thermal wave speed and thermo-elastic coupling traveling with finite speeds and without attenuation. The results are compared with previous results derived by using other generalized thermo-elasticity theories. Numerical results for a hypothetical material are presented.

Keywords: Generalized thermo-elasticity; Without energy dissipation; Distributed heat source

1. Introduction

Problems relating to wave propagation in generalized theories of thermo-elasticity which admit finite speed of thermal signals (second sound effect) in elastic solids have been the subject of active research in recent years. According to these theories, the classical heat transport equation based on Fourier's law becomes fully hyperbolic and thereby the paradox of infinite speed of thermal signals in classical theories is eliminated. Among the generalized theories, the extended thermo-elasticity theory (ETE) proposed by Lord

* Corresponding author. Tel.: +91 342 2623116; fax: +91 342 2530452.
E-mail address: skrc_math@yahoo.com (P.S. Dutta).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2004.12.013

Nomenclature

ρ	constant mass density
τ_{ij}	component of stress tensor
u_i	component of displacement vector
λ, μ	Lame constants
e_{ij}	component of strain tensor
δ_{ij}	Kronecker delta
Δ	dilatation
β	$(3\lambda+2\mu)\alpha_t = 3k\alpha_t$
k	$(\lambda + \frac{2}{3}\mu) =$ bulk modulus
α_t	coefficient of linear thermal expansion
C_V	specific heat at constant strain
T_0	uniform reference temperature
Q	external rate of supply of heat per unit mass
K^*	a material constant characteristic of the theory
C_1	$\sqrt{\frac{(\lambda+2\mu)}{\rho}} =$ dilatational wave velocity
C_3	$\sqrt{\frac{K^*}{\rho C_v}} =$ finite thermal wave speed in TEWOED
C_T	non dimensional thermal wave speed in TEWOED
ε_T	thermo-elastic coupling constant

and Shulman (1967) and the temperature-rate dependent thermo-elasticity theory (TRDTE) developed by Green and Lindsay (1972) have been the subject of recent investigation. Because of the experimental evidence in support of finiteness of heat propagation speed, these generalized thermo-elasticity theories are considered to be more realistic than the conventional thermo-elasticity theory (CTE), particularly in the case of those problems involving very large heat fluxes for short intervals occurring in laser units and energy channels. Several authors like Norwood and Warren (1969), Nayfeh and Nemat-Nasser (1971), Puri (1972), Agarwal (1978), Roychoudhuri and Debnath (1983) and Roychoudhuri (1984, 1985) have studied wave propagation in thermo-elastic and magneto–thermo-elastic solids in the context of ETE and TRDTE.

Problems using CTE concerning an infinite isotropic thermo-elastic solid with distributed time-dependent heat sources have been investigated by Paria (1969). The solutions derived by Paria (1969) consist of a wave part traveling with the speed of modified dilatational wave and a part which is diffusive in nature. Later Roychoudhuri and Bhatta (1981), Roychoudhuri and Sain (1982), Roychoudhuri et al. (1983) investigated the distribution of temperature, displacement, stress and strain in an infinite isotropic solid having instantaneous and continuous heat sources in the context of the ETE and in generalized magneto–thermo-elasticity respectively. In these works small-time solutions were achieved. Later, Sharma (1986) investigated the problem in the context of the TRDTE. The solutions derived by Roychoudhuri and Bhatta (1981), Roychoudhuri and Sain (1982), Roychoudhuri et al. (1983) and Sharma (1986) consist of two waves—one modified dilatational wave and the other modified thermal wave, modified by thermal coupling and thermal relaxation time. The solutions also exhibited discontinuities of the various fields at both the wave fronts and that the discontinuities decay exponentially with distance at the corresponding wave fronts. All the above investigations were carried out in the presence of a thermal field which accommodates dissipation of thermal energy.

Recently, Green and Naghdi (1993) developed a generalized theory of thermo-elasticity which involves thermal displacement gradient as one of the constitutive variables in contrast to the classical coupled thermo-elasticity which includes temperature gradient as one of the constitutive variables. An important feature of this theory is that it does not accommodate dissipation of thermal energy. In this theory the characterization of material response for thermal phenomena is based on three types of constitutive response functions. The nature of those three types of constitutive response functions is such that when the respective theories are linearised, type I is the same as classical heat conduction equation (based on Fourier's law), type II and III admit propagation of thermal signals at finite speeds. But the special characteristic of type II is that it does not accommodate thermal energy dissipation. Several investigations relating to thermo-elasticity without energy dissipation theory (TEWOED) have been presented by Chandrasekharaiah (1996a,b), Chandrasekharaiah and Srinath (1997, 1998), Sharma and Chouhan (1999), and Roychoudhuri and Bandyapadhyay (2004).

The main object of the present paper is to study the thermo-elastic interactions in an isotropic homogeneous infinite thermo-elastic solid containing time-dependent distributed heat sources which vary periodically for a finite time interval in the context of TEWOED of type II. It reveals that the interactions consist of two coupled waves, one following the other and propagating with finite speeds modified by thermal coupling and finite thermal wave speed in TEWOED—one is the predominantly modified elastic wave and other the modified thermal wave and both the waves propagate un-attenuated in contrast to the results derived by Roychoudhuri and Bhatta (1981), Roychoudhuri and Sain (1982), Roychoudhuri et al. (1983) and Sharma (1986) using ETE and TRDTE, where the waves undergo exponential attenuation at the wave fronts. The expressions for displacement, temperature, stress and strain are derived which show that all the results are continuous at both the wave fronts as expected since the heat source is a continuous function of time. Numerical results for a hypothetical material are also presented. The graphical representations indicate that the points of the medium that are beyond the faster wave front do not experience any disturbance. This means that displacement, temperature, stress and strain are identically zero at positions beyond the faster wave front in agreement with the analytical results derived here. This phenomenon has been noticed also in other generalized thermo-elasticity theories such as ETE and TRDTE. The study made in this analysis thus brings to light some similarities and differences between TEWOED, ETE, TRDTE and CTE.

2. Formulation of the problem: basic equations

We consider a homogeneous, thermally conducting isotopic infinite elastic solid at a uniform reference temperature T_0. The infinite solid is also subjected to periodically varying heat sources distributed over a plane area.

The medium is supposed to be unstrained and unstressed initially, but has a uniform reference temperature throughout. It is then subjected to distributed heat sources over the plane $x = 0$ and the solid occupies the whole space $-\infty < x < \infty$.

From the symmetry of the problem the displacement vector $\vec{u}$ has only one component in the x direction so that $\vec{u} = [u(x, t), 0, 0]$ and the temperature increase is $\theta = \theta(x, t)$, where x denotes the spatial co-ordinate and t, the time.

For a homogeneous, isotropic elastic solid, the basic equations for the linear generalized theory of thermo-elasticity of type-II without energy dissipation developed by Green and Naghdi (1993) in absence of body forces are

$$\tau_{ij,j} = \rho \ddot{u}_i (i, j = 1, 2, 3), \tag{1}$$

$$\tau_{ij} = (\lambda \Delta - \beta\theta)\delta_{ij} + 2\mu e_{ij} (i, j = 1, 2, 3) \tag{2}$$

with

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad i, j = 1, 2, 3$$

and

$$\rho C_V \ddot{\theta} + \beta T_0 \mathrm{div}\ddot{\vec{u}} = \rho Q + K^* \nabla^2 \theta \tag{3}$$

with $K^* > 0$.

For the present problem, Eqs. (1)–(3), in one-dimensional case, reduce to

$$C_1^2 \frac{\partial^2 u}{\partial x^2} - \frac{\beta}{\rho}\frac{\partial \theta}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{4}$$

and

$$K^* \frac{\partial^2 \theta}{\partial x^2} + \rho Q = \rho C_V \frac{\partial^2 \theta}{\partial t^2} + \beta T_0 \frac{\partial^3 u}{\partial x \partial t^2}. \tag{5}$$

To transform the above equations in non-dimensional forms, we define the following non-dimensional variables

$$\xi = \frac{x}{l},\ U = \frac{(\lambda + 2\mu)}{\beta T_0 l} u,\ \Theta = \frac{\theta}{T_0},\ \eta = \frac{C_1 t}{l},\ C_{\mathrm{T}}^2 = \frac{K^*}{\rho C_V C_1^2},\ \varepsilon_{\mathrm{T}} = \frac{\beta^2 T_0}{\rho C_V (\lambda + 2\mu)}$$

where $\eta =$ dimensionless time, $l =$ some standard length, $C_{\mathrm{T}} = \frac{C_3}{C_1}$.

Eqs. (4) and (5) in the non-dimensional forms then reduce to

$$\frac{\partial^2 U}{\partial \xi^2} - \frac{\partial \Theta}{\partial \xi} = \frac{\partial^2 U}{\partial \eta^2} \tag{6}$$

and

$$\frac{\partial^2 \Theta}{\partial \eta^2} + \varepsilon_{\mathrm{T}} \frac{\partial^3 U}{\partial \xi \partial \eta^2} = Q_0 + C_{\mathrm{T}}^2 \frac{\partial^2 \Theta}{\partial \xi^2}, \tag{7}$$

where $Q_0 = \frac{Q l^2}{C_V T_0 C_1^2}$.

For heat sources distributed over the plane $x = 0$, we may represent it as

$$Q_0 = Q_0^* \delta(\xi) \sin\left(\frac{\pi \eta}{\tau}\right), \quad \text{for } 0 \leqslant \eta \leqslant \tau \quad \text{and} \quad 0 \quad \text{for } \eta > \tau.$$

Here Q_0^* is a constant and $\delta(\xi)$ is Dirac's delta function defined by

$$\int_{-\infty}^{+\infty} \delta(\xi) \mathrm{d}\xi = 1; \quad \text{and} \quad \delta(\xi) = 0 \quad \text{for } \xi \neq 0.$$

Finally, we obtain the governing equations in TEWOED of type-II (in absence of body forces) in non-dimensional forms as

$$\frac{\partial^2 U}{\partial \xi^2} - \frac{\partial \Theta}{\partial \xi} = \frac{\partial^2 U}{\partial \eta^2} \tag{8}$$

and

$$\frac{\partial^2 \Theta}{\partial \eta^2} + \varepsilon_{\mathrm{T}} \frac{\partial^3 U}{\partial \xi \partial \eta^2} = Q_0^* \delta(\xi) \sin\left(\frac{\pi \eta}{\tau}\right) + C_{\mathrm{T}}^2 \frac{\partial^2 \Theta}{\partial \xi^2} \tag{9}$$

3. Application of Laplace and Fourier transforms, solution of the problem in transformed domain

Applying Laplace transform defined by

$$\overline{\phi}(\xi,p)=\int_0^{\infty}\phi(\xi,\eta)e^{-p\eta}\mathrm{d}\eta, Re(p)>0$$

with respect to time variable η and then the complex Fourier transform defined by

$$\overline{\phi_1}(\alpha,p)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\overline{\phi}(\xi,p)\mathrm{e}^{i\alpha\xi}\mathrm{d}\xi$$

with respect to space variable ξ to Eqs. (8) and (9), we obtain

$$(\alpha^2+p^2)\overline{U_1}(\alpha,p)=i\alpha\overline{\Theta_1}(\alpha,p), \tag{10}$$

$$(C_{\mathrm{T}}^2\alpha^2+p^2)\overline{\Theta_1}(\alpha,p)-i\alpha\varepsilon_{\mathrm{T}}p^2\overline{U_1}(\alpha,p)=\frac{Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{\sqrt{2\pi}(\pi^2+p^2\tau^2)}. \tag{11}$$

Solving (10) and (11) for $\overline{U_1}(\alpha,p)$ and $\overline{\Theta_1}$ (α,p), we obtain

$$\overline{U_1}(\alpha,p)=\frac{i\alpha Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{(\pi^2+p^2\tau^2)M^*\sqrt{2\pi}}, \tag{12}$$

$$\overline{\Theta_1}(\alpha,p)=\frac{Q_0^*\pi\tau(1+e^{-p\tau})(\alpha^2+p^2)}{(\pi^2+p^2\tau^2)M^*\sqrt{2\pi}}, \tag{13}$$

where

$$M^*=(\alpha^2+p^2)(C_{\mathrm{T}}^2\alpha^2+p^2)+\alpha^2p^2\varepsilon_{\mathrm{T}}=C_{\mathrm{T}}^2\alpha^4+(1+C_{\mathrm{T}}^2+\varepsilon_{\mathrm{T}})\alpha^2p^2+p^4=C_{\mathrm{T}}^2(\alpha^2+K_1)(\alpha^2+K_2).$$

Here $K_1+K_2=(\frac{1+C_{\mathrm{T}}^2+\varepsilon_{\mathrm{T}}}{C_{\mathrm{T}}^2})p^2$ and $K_1K_2=\frac{p^4}{C_{\mathrm{T}}^2}$,

K_1, K_2 are the roots of the equation

$$\left(\alpha^4-\frac{1+C_{\mathrm{T}}^2+\varepsilon_{\mathrm{T}}}{C_{\mathrm{T}}^2}p^2\alpha^2+\frac{p^4}{C_{\mathrm{T}}^2}\right)=0.$$

Setting $K_i=\lambda_i\mathrm{p}^2$, $i=1$, 2, we have

$$\lambda_1+\lambda_2=\frac{1+C_{\mathrm{T}}^2+\varepsilon_{\mathrm{T}}}{C_{\mathrm{T}}^2}=A>0,$$

$$\lambda_1-\lambda_2=\frac{\sqrt{\left(C_{\mathrm{T}}^2-1\right)^2+2\left(C_{\mathrm{T}}^2+1\right)\varepsilon_{\mathrm{T}}+\varepsilon_{\mathrm{T}}^2}}{C_{\mathrm{T}}^2}=\frac{\sqrt{\Gamma}}{C_{\mathrm{T}}^2}=B>0,$$

where

$$\Gamma=\left\lfloor(C_{\mathrm{T}}^2-1)^2+2(C_{\mathrm{T}}^2+1)\varepsilon_{\mathrm{T}}+\varepsilon_{\mathrm{T}}^2\right\rfloor>0. \tag{13.1}$$

Here $\lambda_1=\frac{A+B}{2}$, $\lambda_2=\frac{A-B}{2}$ and both are real and

$$K_1=\frac{A+B}{2}p^2=N_1^2p^2,\quad K_2=\frac{A-B}{2}p^2=N_2^2p^2$$

where

$$N_{1,2}^2 = \frac{A \pm B}{2} = \frac{1}{2C_T^2}\left[\left(1 + C_T^2 + \varepsilon_T\right) \pm \sqrt{\Gamma}\right]. \tag{13.2}$$

Again using dimensionless quantities, we obtain

$$\sigma(\xi,\eta) = \frac{\tau_{xx}}{\beta T_0} = \frac{\partial U}{\partial \xi} - \Theta \tag{14}$$

and

$$\varepsilon(\xi,\eta) = \frac{\beta T_0}{(\lambda+2\mu)}\frac{\partial U}{\partial \xi} = M_1\frac{\partial U}{\partial \xi}, \text{ here } M_1 = \frac{\beta T_0}{(\lambda+2\mu)}, \tag{15}$$

where $\sigma(\xi,\eta)$ and $\varepsilon(\xi,\eta)$ are respectively the non-dimensional stress and strain.

First by applying Laplace transform on time η and then Fourier transform on space variable ξ, we obtain

$$(\overline{\sigma_1})(\alpha,p) = -i\alpha\overline{U}_1(\alpha,p) - \overline{\Theta}_1(\alpha,p) = -i\alpha\frac{i\alpha Q_0^*\pi\tau(1+e^{-p\tau})}{(\pi^2+p^2\tau^2)M^*\sqrt{2\pi}} - \frac{Q_0^*\pi\tau(1+e^{-p\tau})(\alpha^2+p^2)}{(\pi^2+p^2\tau^2)M^*\sqrt{2\pi}} \tag{16}$$

and

$$\overline{\varepsilon_1}(\alpha,p) = -i\alpha M_1\overline{U}_1(\alpha,p) = M_1\frac{\alpha^2 Q_0^*\pi\tau(1+e^{-p\tau})}{(\pi^2+p^2\tau^2)M^*\sqrt{2\pi}} \tag{17}$$

Now inverse Fourier transform gives the following solutions in the Laplace transform domain of Eqs. (12), (13), (16) and (17) for displacement, temperature, stress and strain as

$$\overline{U}(\xi,p) = \frac{Q_0^*\pi\tau}{2C_T^2\left(N_2^2-N_1^2\right)}\left[\frac{e^{-\xi N_1 p}(1+e^{-p\tau})}{p^2(\pi^2+p^2\tau^2)} - \frac{e^{-\xi N_2 p}(1+e^{-p\tau})}{p^2(\pi^2+p^2\tau^2)}\right] \quad \text{for } \xi > 0, \tag{18.1}$$

$$\overline{\Theta}(\xi,p) = \frac{Q_0^*\pi\tau}{2C_T^2\left(N_2^2-N_1^2\right)}\left[\frac{(1-N_1^2)}{N_1}\frac{e^{-\xi N_1 p}(1+e^{-p\tau})}{p(\pi^2+p^2\tau^2)} - \frac{(1-N_2^2)}{N_2}\frac{e^{-\xi N_2 p}(1+e^{-p\tau})}{p(\pi^2+p^2\tau^2)}\right] \quad \text{for } \xi > 0, \tag{18.2}$$

$$\overline{\sigma}(\xi,p) = \frac{Q_0^*\pi\tau}{2C_T^2\left(N_1^2-N_2^2\right)}\left[\frac{1}{N_1}\frac{e^{-\xi N_1 p}(1+e^{-p\tau})}{p(\pi^2+p^2\tau^2)} - \frac{1}{N_2}\frac{e^{-\xi N_2 p}(1+e^{-p\tau})}{p(\pi^2+p^2\tau^2)}\right] \quad \text{for } \xi > 0 \tag{18.3}$$

and

$$\overline{\varepsilon}(\xi,p) = \frac{M_1 Q_0^*\pi\tau}{2C_T^2\left(N_1^2-N_2^2\right)}\left[N_1\frac{e^{-\xi N_1 p}(1+e^{-p\tau})}{p(\pi^2+p^2\tau^2)} - N_2\frac{e^{-\xi N_2 p}(1+e^{-p\tau})}{p(\pi^2+p^2\tau^2)}\right] \quad \text{for } \xi > 0. \tag{18.4}$$

Now inverse Laplace transforms of expressions (18.1)–(18.4) give the closed form solutions for displacement, temperature, stress and strain as

$$\begin{aligned} U(\xi,\eta) = {} & M\left[\left\{(\eta-\xi N_1) - \frac{\tau}{\pi}\sin\frac{\pi}{\tau}(\eta-\xi N_1)\right\}H(\eta-\xi N_1)\right] \\ & + M\left[\left\{(\eta-\xi N_1-\tau) + \frac{\tau}{\pi}\sin\frac{\pi}{\tau}(\eta-\xi N_1)\right\}H(\eta-\xi N_1-\tau)\right] \\ & - M\left[\left\{(\eta-\xi N_2) - \frac{\tau}{\pi}\sin\frac{\pi}{\tau}(\eta-\xi N_2)\right\}H(\eta-\xi N_2)\right] \\ & - M\left[\left\{(\eta-\xi N_2-\tau) + \frac{\tau}{\pi}\sin\frac{\pi}{\tau}(\eta-\xi N_2)\right\}H(\eta-\xi N_2-\tau)\right], \end{aligned} \tag{19.1}$$

$$\Theta(\xi,\eta)=\frac{M(1-N_1^2)}{N_1}\left[\left\{1-\cos\frac{\pi}{\tau}(\eta-\xi N_1)\right\}H(\eta-\xi N_1)+\left\{1+\cos\frac{\pi}{\tau}(\eta-\xi N_1)\right\}H(\eta-\xi N_1-\tau)\right]$$
$$-\frac{M(1-N_2^2)}{N_2}\left[\left\{1-\cos\frac{\pi}{\tau}(\eta-\xi N_2)\right\}H(\eta-\xi N_2)+\left\{1+\cos\frac{\pi}{\tau}(\eta-\xi N_2)\right\}H(\eta-\xi N_2-\tau)\right], \tag{19.2}$$

$$\sigma(\xi,\eta)=-\frac{M}{N_1}\left[\left\{1-\cos\frac{\pi}{\tau}(\eta-\xi N_1)\right\}H(\eta-\xi N_1)+\left\{1+\cos\frac{\pi}{\tau}(\eta-\xi N_1)\right\}H(\eta-\xi N_1-\tau)\right]$$
$$+\frac{M}{N_2}\left[\left\{1-\cos\frac{\pi}{\tau}(\eta-\xi N_2)\right\}H(\eta-\xi N_2)+\left\{1+\cos\frac{\pi}{\tau}(\eta-\xi N_2)\right\}H(\eta-\xi N_2-\tau)\right] \tag{19.3}$$

and

$$\varepsilon(\xi,\eta)=-MM_1N_1\left[\left\{1-\cos\frac{\pi}{\tau}(\eta-\xi N_1)\right\}H(\eta-\xi N_1)+\left\{1+\cos\frac{\pi}{\tau}(\eta-\xi N_1)\right\}H(\eta-\xi N_1-\tau)\right]$$
$$+MM_1N_2\left[\left\{1-\cos\frac{\pi}{\tau}(\eta-\xi N_2)\right\}H(\eta-\xi N_2)+\left\{1+\cos\frac{\pi}{\tau}(\eta-\xi N_2)\right\}H(\eta-\xi N_2-\tau)\right]. \tag{19.4}$$

Here $M=\dfrac{Q_0^*\tau}{2\pi C_T^2(N_2^2-N_1^2)}$.

4. Numerical results and discussion

The closed form solutions for displacement, temperature, stress and strain reveal the existence of two coupled waves. The terms associated with $H(\eta-\xi N_1)$ and $H(\eta-\xi N_1-\tau)$ represent the contribution of the wave traveling with speed $V_E=1/N_1$ at the wave front $\xi=V_E.t$ at two different times $t=\eta$ and $\eta-\tau$. The terms associated with $H(\eta-\xi N_2)$ and $H(\eta-\xi N_2-\tau)$ represent the contribution of wave traveling with speed $V_T=1/N_2$ at the wave front $\xi=V_T.t$ at two different times $t=\eta$ and $\eta-\tau$, where

$$V_E,V_T=\frac{1}{N_{1,2}}=\frac{\sqrt{2}C_T}{\sqrt{[(1+C_T^2+\varepsilon_T)\pm\sqrt{\Gamma}]}}. \tag{20}$$

From solutions (19.1)–(19.4) we thus observe that each of $U(\xi,\eta)$, $\Theta(\xi,\eta)$, $\sigma(\xi,\eta)$ and $\varepsilon(\xi,\eta)$ is made up of two parts and that each part corresponds to a wave propagating with finite speed, the speed of the wave corresponding to first part being $V_E=1/N_1$ and that corresponding to second part being $V_T=1/N_2$. Using the expressions (13.1), (13.2) and (20), we observe that

(i) $V_E<V_T$.
(ii) For material in which $K^*>\rho C_V C_1^2(C_T>1)$, $V_E\to 1$ (unit dilatational wave speed) and $V_T\to C_T$ (finite thermal wave speed) when $\varepsilon_T\to 0$. Thus when $\varepsilon_T\neq 0$, V_E and V_T correspond respectively to the modified elastic wave (e-wave) and the modified thermal wave (θ-wave). The faster wave is predominantly the θ-wave and the slower wave is e-wave.
(iii) Further for material in which $K^*<\rho C_V C_1^2(C_T<1)$, $V_E\to C_T$ (finite thermal wave speed) and $V_T\to 1$ (unit dilatational wave speed) when $\varepsilon_T\to 0$. Thus when $\varepsilon_T\neq 0$, V_E and V_T correspond respectively to the modified thermal wave (θ-wave) and the modified elastic wave (e-wave). The faster wave is predominantly the e-wave and the slower wave is θ-wave.

This concludes that the disturbances consist of two coupled waves, one following the other. From the solutions (19.1)–(19.4), we observe two more interesting features. First, neither the e-wave nor the θ-wave experiences any decay with distance (attenuation). Secondly, all of $U(\xi,\eta)$, $\Theta(\xi,\eta)$, $\sigma(\xi, \eta)$ and $\varepsilon(\xi,\eta)$ are identically zero for $\xi > V_T.\eta$, where V_T is the speed of faster wave. This implies that at a given instant of time $\eta^*(>0)$, the points of the solid that are beyond the faster wave front do not experience any disturbance. Further the solutions in the context of ETE (L–S theory) and TRDTE (G–L theory) indicate that both e-wave and θ-wave decay exponentially with distance. This difference between the predictions of TEWOED and those of ETE and TRDTE stems from the fact that while TEWOED does not sustain energy dissipation, ETE and TRDTE both do accommodate energy dissipation due to the presence of temperature-rate term in the heat transport equation.

With an aim to illustrate the problem numerically, we choose $\varepsilon_T = 0.073$ (Calcium epoxy). Fig. 1 illustrates the graphs of V_E, V_T versus C_T. Dotted lines show the variation of V_E and V_T for $\varepsilon_T = 0$. The graphs show that $V_E = 1$ for $\varepsilon_T = 0$. For $\varepsilon_T = 0.073$, the thick lines represent the variations of V_E and V_T versus C_T. Clearly the graph shows that $V_T > V_E$ implying that modified elastic wave follows the modified thermal wave for $C_T > 1$. For $C_T < 1$, the faster wave speed is referred to as V_E and the slower wave as $V_T(V_E > V_T)$.

Next we consider a copper like material for which $\varepsilon_T = 0.0168$, and take the representative values $T_0 = 1$, $Q_0^* = 1$, $\tau = 1$, $\lambda = 1.387 \times 10^{12}$ dyne/cm^2, $\mu = 0.448 \times 10^{12}$ dyne/cm^2, $\alpha_t = 1.67 \times 10^{-8}$/°C, $C_T > 1$ (the faster wave front happens to be the θ-wave front).

In case of TEWOED we choose $C_T = 2$, as a test example so that modified e-wave follows the modified θ-wave. Figs. 2–5 exhibits the variation of displacement, temperature, stress and strain versus distance ξ from which we make the following observations:

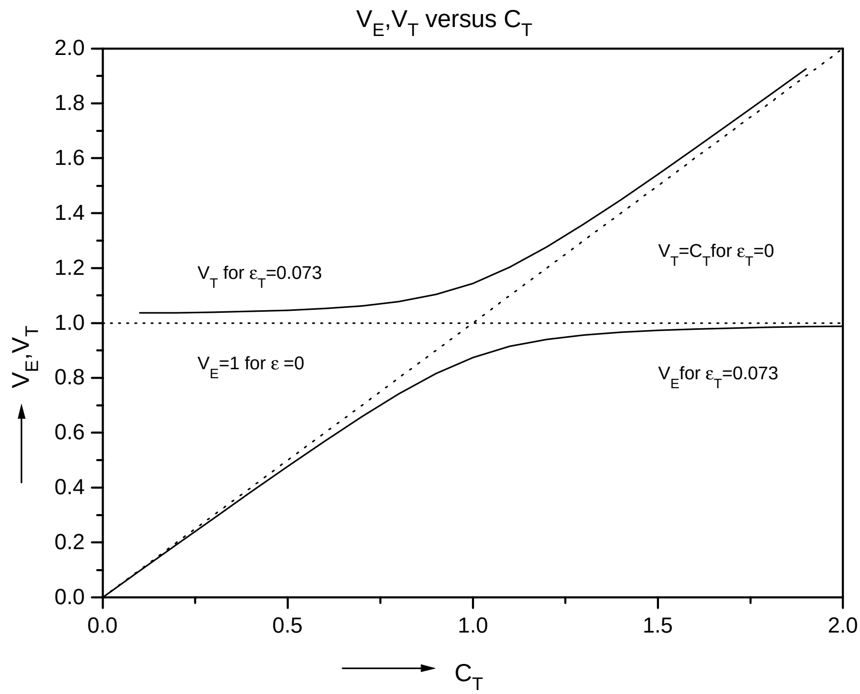

Fig. 1. V_E, V_T versus C_T.

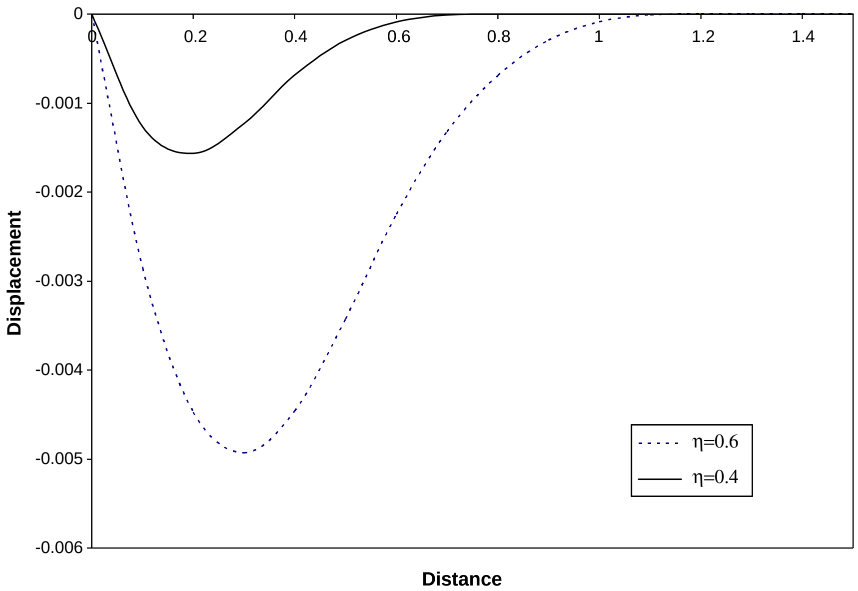

Fig. 2. Displacement $U(\xi,\eta)$ versus distance ξ.

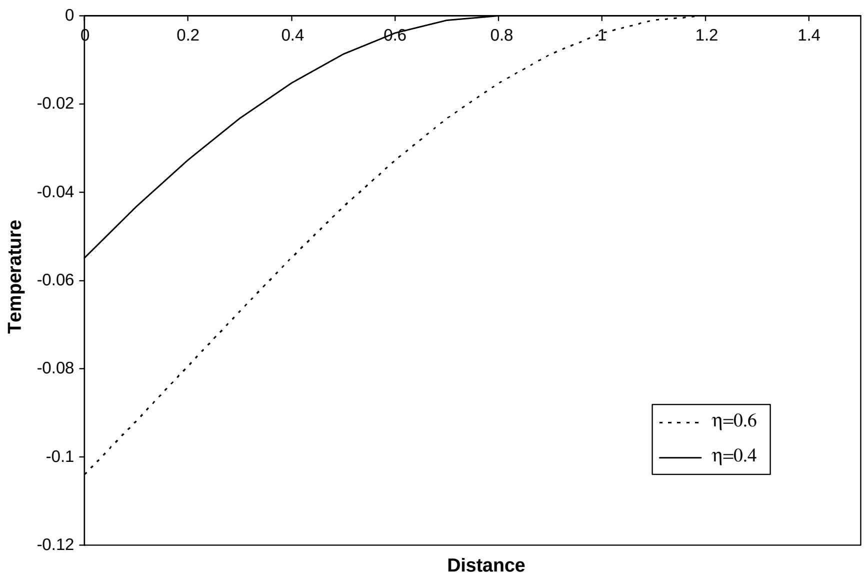

Fig. 3. Temperature $\Theta(\xi,\eta)$ versus distance ξ.

Fig. 2 represents variation of displacement versus distance. The graph shows negative values of displacement which means that it is in opposite direction. This is because the heat source varies periodically with time (a sine pulse active for a short duration). The magnitude of the displacement first increases very rapidly with distance, attains a maximum value and then gradually decreases to zero values for $\xi \geqslant 0.8$ (when $\eta = 0.4$) and $\xi \geqslant 1.2$ (when $= 0.6$). This is because that the thermal wave front is positioned at $\xi = 0.8$

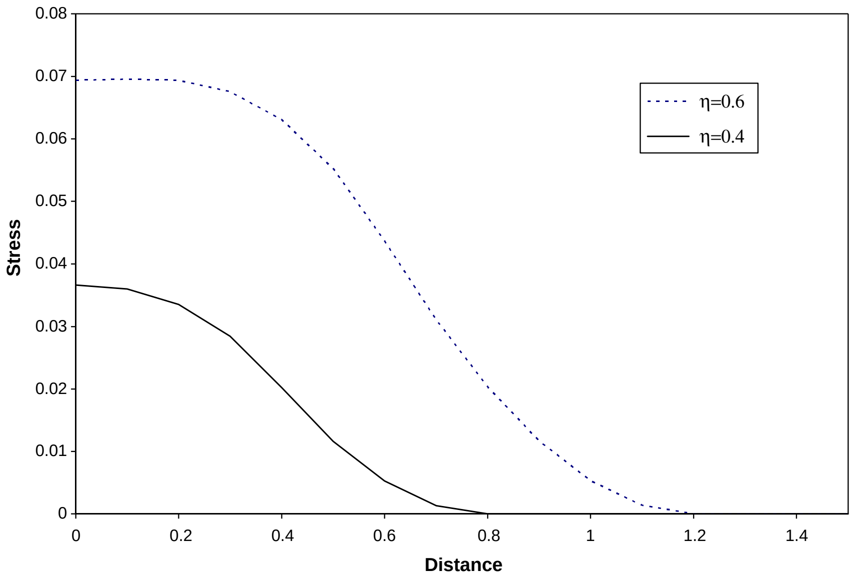

Fig. 4. Stress $\sigma(\xi,\eta)$ versus distance ξ.

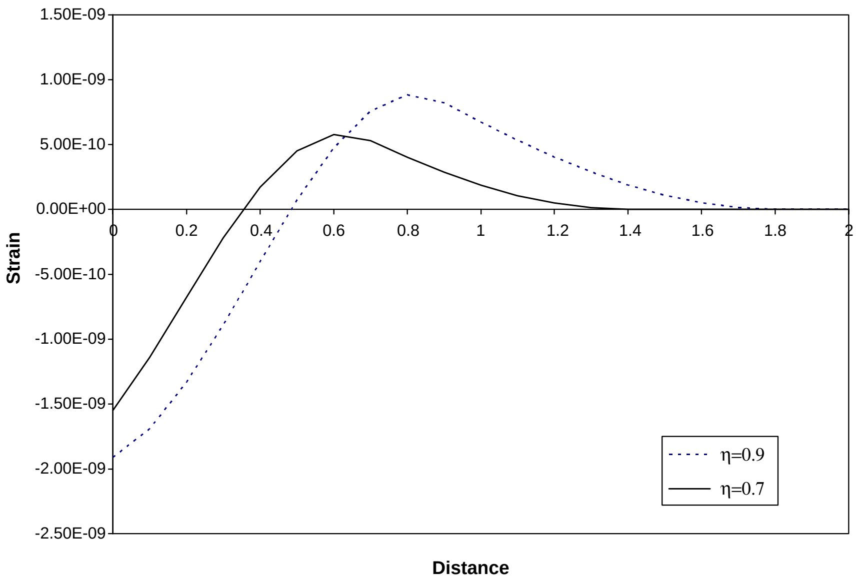

Fig. 5. Strain $\varepsilon(\xi,\eta)$ versus distance ξ.

(at the instant $\eta = 0.4$) and $\xi = 1.2$ (at the instant $\eta = 0.6$) and beyond this wave front, the disturbance vanishes.

Fig. 3 gives the temperature distribution. The graph shows negative values of Θ, which, in fact refers to the small temperature change (Chandrasekharaiah and Srinath, 1997). It is decreasing in nature and finally vanishes beyond the thermal wave front located at $\xi = 0.8$ (when $\eta = 0.4$) and $\xi = 1.2$ (when $\eta = 0.6$) in agreement with the theoretical results.

Fig. 4 represents variation of stress versus distance. Its magnitude decreases with the increase of distance and finally goes to zero. $\sigma(\xi, \eta)$ lies in the domain $0 \leqslant \xi \leqslant 0.8$ (for $\eta = 0.4$) and $0 \leqslant \xi \leqslant 1.2$ (for $\eta = 0.6$). This is also in agreement with the fact that stress should diminish with increasing distance from the plane $\xi = 0$ and that beyond the faster wave front, disturbance must vanish.

Fig. 5 represents variation of strain versus distance. The figure shows negative values for strains in the range $0 \leqslant \xi \leqslant 0.35$ (for $\eta = 0.7$) and $0 \leqslant \xi \leqslant 0.45$ (for $\eta = 0.9$) and positive values in the range $0.35 \leqslant \xi \leqslant 1.4$ (for $\eta = 0.7$) and $0.45 \leqslant \xi \leqslant 1.8$ (for $\eta = 0.9$) and then finally diminishes. The strain distribution after assuming positive values goes on increasing, attains maximum values and then decreases slowly and finally vanishes. This is also in conformity with the fact that strain should decrease with increasing distance ξ from the plane $\xi = 0$ where heat source is active for a very short duration. The analytical solutions for displacement, temperature, stress and strain are identically zero for $\xi > V_{\mathrm{T}} \cdot \eta$ because of the fact that the points of the solid that are beyond the faster wave front $\xi = V_{\mathrm{T}} \cdot \eta$ do not experience any disturbance. This is also in agreement with graphical representation (Figs. 2–5), showing location of the faster wave front at two different instants of time beyond which all disturbances vanish in agreement with the theoretical result thus obtained.

5. Concluding remarks

From the above results we conclude the following points:

(a) We have to choose the value of K^* (a material constant characteristic of TEWOED), depending on C_{T} in such a way that in wave propagation problem in TEWOED it agrees with the inequality $V_{\mathrm{E}} < V_{\mathrm{T}}$ for those material for which $C_{\mathrm{T}} > 1$ and $V_{\mathrm{E}} > V_{\mathrm{T}}$ for those material for which $C_{\mathrm{T}} < 1$.
(b) The points of the medium, at a given instant of time, that lie beyond the faster wave front do not experience any disturbance. This phenomenon is a characteristic of all the generalized thermo-elasticity theories. Our observations thus verify that TEWOED is really a generalized thermo-elasticity theory. Further the waves do not experience any attenuation in contrast to the other thermo-elasticity theories. The present study thus brings to light some similarity and differences between TEWOED, ETE, TRDTE and CTE.

Acknowledgments

The authors express their deep gratitude and thanks to the respected reviewers for their helpful suggestions. This work was supported by the University of Burdwan.

References

Agarwal, V.K., 1978. On surface waves in generalized thermo-elasticity. Journal of Elasticity 8, 171–177.

Chandrasekharaiah, D.S., 1996a. A note on the uniqueness of solution in the linear theory of thermo-elasticity without energy dissipation. Journal of Elasticity 43, 279–283.

Chandrasekharaiah, D.S., 1996b. Thermo-elastic plane waves without energy dissipation. Mechanics Research Communication 23, 549–555.

Chandrasekharaiah, D.S., Srinath, K.S., 1997. Axisymmetric thermo-elastic interaction without energy dissipation in an unbounded body with cylindrical cavity. Journal of Elasticity 46, 19–31.

Chandrasekharaiah, D.S., Srinath, K.S., 1998. Thermo-elastic interactions without energy dissipation due to a point heat sources. Journal of Elasticity 50, 97–108.

Green, A.E., Lindsay, K.A., 1972. Thermo-elasticity. Journal of Elasticity 2, 1–7.

Green, A.E., Naghdi, P.M., 1993. Thermo-elasticity without energy dissipation. Journal of Elasticity 31, 189–208.

Lord, H., Shulman, Y., 1967. A generalized dynamical theory of thermo-elasticity. Journal of Mechanics, Physics and Solids 15, 299–309.

Nayfeh, A.H., Nemat-Nasser, S., 1971. Thermo-elastic waves in solids with thermal relaxation. Acta Mechanica 12, 53–69.

Norwood, F.R., Warren, W.E., 1969. Wave propagation in the generalized dynamical theory of thermo-elasticity. Quarterly Journal of Mechanics and Applied Mathematics XXII, 283–290, pt-3.

Paria, G., 1969. Instantaneous heat sources in an infinite elastic solid. Indian Journal of Mechanics and Mathematics. Pt-2, Special Issue, 41–50.

Puri, P., 1972. Plane waves in thermo-elasticity and magneto–thermo-elasticity. International Journal of Engineering Science 10, 467–476.

Roychoudhuri, S.K., 1984. Electro-magneto–thermo-elastic plane waves in rotating media with thermal relaxation. International Journal of Engineering Science 22, 519–530.

Roychoudhuri, S.K., 1985. Effect of rotation and relaxation times on plane waves in generalized thermo-elasticity. Journal of Elasticity 15, 59–68.

Roychoudhuri, S.K., Bandyapadhyay, N., 2004. Thermo-elastic wave propagation in a rotating elastic medium without energy dissipation. International Journal of Mathematics and Mathematical Sciences, in press.

Roychoudhuri, S.K., Bhatta, N., 1981. Thermo-elastic interactions in an infinite elastic solid with thermal relaxation. Proceedings of National Academy of Sciences India 47-A, pt-A, Physical Sciences 4, 499–510.

Roychoudhuri, S.K., Debnath, L., 1983. Magneto–thermo-elastic plane waves in rotating media. International Journal of Engineering Science 21, 155–163.

Roychoudhuri, S.K., Sain, G.D., 1982. Instantaneous heat sources in an infinite elastic solid with thermal relaxation. Indian Journal of Pure and Applied Mathematics 13, 1340–1353.

Roychoudhuri, S.K., Sain, G.D., Bhatta, N., 1983. Magneto–thermo-elastic interaction in an infinite conducting elastic solid due to heat sources with thermal relaxation. Proceedings of National Academy of sciences India. 49, 602–621.

Sharma, J.N., 1986. Instantaneous heat sources in temperature rate dependent thermo-elasticity. Proceedings of National Academy of sciences India 52-A, 1375–1383.

Sharma, J.N., Chouhan, R.S., 1999. On the problem of body forces and heat sources in thermo-elasticity without energy dissipation. Indian Journal of Pure and Applied Mathematics 30, 595–610.